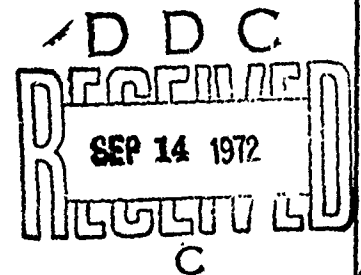


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DIFFRACTION OF WAVES ON AN UNEVEN SURFACE, II.

by
L. M. Brekhovskikh

USSR

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Several specific instances of the diffraction of acoustic and electromagnetic waves on wavy surfaces of various types are examined on the basis of general theory. Here the finite nature of the dimensions of the area on which the diffraction occurs is taken into account. In the case where the uneven condition is sinusoidal, the results obtained by two different methods are compared.

In work [1] (we shall designate it 1 in the future) an approximate theory of the diffraction of acoustic and electromagnetic waves on an uneven surface, which had uneven areas which were large in comparison to the length of the wave, on the assumption that the surface was infinitely extended, but that the uneven conditions were periodic, was given. We shall consider the finite nature of the surface, as well as examine several specific examples, below.

1. Diffraction on an Uneven Surface of Limited Dimensions

Let a limited sector on an uneven surface participate in the scattering of waves. This may take place both as a consequence of the limited nature of the area and as a consequence of the limited nature of its illuminated sector. For the sake of simplicity, we shall examine here a two-dimensional problem, i.e., we shall assume that the "active" sector of the uneven surface has the form of an infinite strip positioned in plane xy perpendicularly to the wave incidence plane xz . As in [1] we shall describe the unevenness as a function depending, in the case under examination, only on one variable X . We shall assume the width of the area to be large in comparison to the period of unevenness Λ_x .

For an example, we shall examine the potential of the acoustic field of a scattered wave, which in the case of an infinite area is given by the integral (12,1)¹ we shall introduce as an integral the

¹ In the case of an electromagnetic wave, the integral (32,1) is equivalent to it.

function $F(X)$, which characterizes the change in the "illumination" of the area in direction X . As a result, if one considers that we are examining a plane problem and the double Fourier series is reduced to a single series, we shall obtain

$$\rho = \frac{1}{(2\pi)^2} \sum_m \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} B_m(k_x, k_y, k_z) F(X) \exp \{i(k_x x + k_y y + k_z z)\} \times \\ \times \exp \{i(k_x^0 - k_x + mp)X + i(nq - k_y)Y\} dX dY dk_x dk_y. \quad (1)$$

If the area is illuminated or sonicated with a sharply limited beam with a constant intensity throughout its cross-section, or the illumination is accomplished by an infinite plane wave, but the area itself has the form of a strip beyond the limits of which there is no reflection, $F(X)$ will be equal to 1 within the strip and zero outside of it. Generally, the illumination function $F(X)$ is more complex. Here we shall consider it to be little changing over the wave length of unevenness Λ_x .

Integration in (1) for Y and k_y is accomplished in the same manner as in § 2 and 3 (see 1) for an infinite area, which is natural, insofar as the area is assumed to be infinite in direction y , as was previously the case. As a result we shall obtain

$$\rho = \frac{1}{2\pi} \sum_m \int_{-\infty}^{+\infty} B(k_x, k_z) \exp \{i(k_x x + k_z z)\} dk_x \times \\ \times \int_{-\infty}^{+\infty} F(X) \exp \{i(k_x^0 - k_x + mp)X\} dX. \quad (2)$$

We shall write the expansion of the function $F(X)$ in the Fourier integral:

$$F(X) = \int_{-\infty}^{+\infty} e^{-ik_x X} \Phi(x) dx, \quad (3)$$

where $\Phi(x)$ is the function characterizing the spectral expansion density defined by the equality:

$$\Phi(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ik_x X} F(X) dX. \quad (4)$$

In Fourier integral theory a relationship is known which is analogous to the relationship of indeterminacy, according to which intervals D and Δx of the changes in variables X and x , where the functions $F(X)$ and $\Phi(x)$ have values notably different than zero, are related by the relationship:

$$D\Delta x \sim 2\pi. \quad (5)$$

Since we are suggesting that $D \gg \Lambda_x$ (many periods of sinusoidality are being illuminated), the latter relationship yields

$$\Delta x \ll 2\pi/\Lambda = p, \quad (6)$$

which we shall utilize in the future.

Besides that, if $F(X)$ is a function having a single maximum in the middle of the area, then it follows from (4) that $\Phi(x)$ will be maximum when $x = 0$. Comparing (2) and (4), we find

$$\varphi = \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} B_m(k_x, k_z) \Phi(k_x^0 - k_x + mp) \exp\{i(k_x x + k_z z)\} dk_x. \quad (7)$$

From here it is evident that in the case of a limited area there are scattered waves with arbitrary k_x , i.e., waves distributing themselves in all directions. However, for each m the amplitudes of these waves differ notably from zero only within the limits of

$$k_x = k_x^0 + mp \pm \Delta x,$$

since beyond these limits the function $\Phi(k_x^0 - k_x + mp)$ turns out to be practically equal to zero. Since according to (6) $\Delta x \ll p$, it consequently follows that the scattered waves will group themselves in narrow angular intervals close to the directions set by the equation

$$k_x = k_x^0 + mp, \quad m = 0, \pm 1, \pm 2, \dots$$

We shall examine the field at sufficiently great distances from the area, particularly in the Fraunhofer zone. We shall make the beginning of the coordinates and the center of the area coincident and we shall define the angle created by the direction to the point of observation and the x axis as α . Obviously,

$$x = R \cos \alpha, \quad z = R \sin \alpha,$$

where R is the distance from the center of the area to the point of observation. The exponential curve in (7) will now be written:

$$\exp \{iR(k_x \cos \alpha + \sqrt{k^2 - k_x^2} \sin \alpha)\}.$$

Since R is assumed to be large, when k_x changes the exponential curve will be an extremely rapidly changing function. This makes it possible to use either the stationary phase [2] or the saddle-point [3] method to calculate the integral. In this case the principal input to the integral will be only the values of k_x lying near the saddle points $k_x = v$, which are determined from the equation

$$[(\partial/\partial k_x)(k_x \cos \alpha + \sqrt{k^2 - k_x^2} \sin \alpha)]_{k_x=v} = 0,$$

from whence

$$v = k \cos \alpha.$$

To calculate the integral in this case, in the first approximation the entire subintegral function, except the exponential curve, may be moved beyond the integral sign when $k_x = v$; the expression in the exponential curve may be expanded in a series according to the powers of $k_x - v$. As a result, we obtain from (7):

$$\begin{aligned} \varphi = & \sum_{m=-\infty}^{+\infty} B_m(\alpha) \Phi(k_x^0 + mp - k \cos \alpha) e^{ikR} \times \\ & \times \int_{-\infty}^{+\infty} \exp \left\{ -\frac{i}{k} R (k_x - v)^2 \frac{1}{\sin^2 \alpha} \right\} dk_x. \end{aligned} \quad (8)$$

Here $B_m(k_x, k_z)$ is designated by $B_m(\alpha)$ when $k_x = k \cos \alpha$ and $k_z = k \sin \alpha$; the integral in this case is reduced to the well-known Poisson integral by substituting $k_x - v = e^{-i\pi/4} \bullet$ and it yields

$$\sqrt{\pi k/R} \sin \alpha e^{-i\pi/4}.$$

In view of the fact that the function $\Phi(k_x^0 + mp - k \cos \alpha)$ differs from zero only in the narrow interval where $\Delta x \ll p$, one may only consider, of all the terms of the sum in (8), only that term for which m is equal to a whole number close to the value:

$$(1/p)(k \cos \alpha - k_x^0). \quad (9)$$

As a result, we obtain:

$$\varphi = B_m(\alpha) \sin \alpha \Phi(k_x^0 + mp - k \cos \alpha) \exp \{ - (i\pi/4) + ikR \} \sqrt{\pi k/R}. \quad (10)$$

Thus the diffraction field in this case will be a cylindrically diverging wave of a directional nature in the form of separate lobes, the maxima of which lie in the directions of $\alpha = \alpha_m$, which are determined by the equality:

$$k \sin \alpha_m = k_x^0 + mp, \quad m = 0, \pm 1, \pm 2, \dots \quad (11)$$

In these directions the argument of the function Φ becomes zero, and the function itself is maximum. The width of the lobes is determined by the magnitude of the interval $\Delta \alpha \approx 2\pi/D$, in which the function is notably different from zero [see (4) and (5)]. In conformity with this, the boundary of lobe m may be defined as angle α'_m , for which

$$k_x^0 + mp - k \cos \alpha'_m \approx 2\pi/D. \quad (12)$$

From (11) and (12) we obtain

$$k(\cos \alpha_m - \cos \alpha'_m) \approx 2\pi/D;$$

from whence, considering the closeness of the values of angles α_m and α'_m ,

$$\alpha_m - \alpha'_m \approx \lambda/D \sin \alpha_m, \quad (13)$$

which will be the angular half-width of each lobe. The number of lobes is determined by the interval of whole values of m where $|\cos \alpha_m| \leq 1$ and approximately equal to $2A/\lambda$. The results obtained do not differ in principle from the results obtained during the investigation of diffraction from an optical grid of finite dimensions [4]. If we were to have examined diffraction from an area bounded in both directions, we should have obtained results analogous to those presented in [4] (§ 52) for a two-dimensional grid.

The form of each lobe is determined by the form of function Φ in (10), and, consequently, according to (4), the form of function $F(X)$. In the case of a limited area with illumination which is constant over the entire area, we have

$$\begin{aligned} F(X) &= 1, \quad -D/2 \leq X \leq D/2, \\ F(X) &= 0, \quad |X| > D/2, \end{aligned} \quad (14)$$

and from (4) we find that

$$\Phi(x) = \frac{1}{\pi x} \sin \frac{x D}{2}. \quad (15)$$

For the maximum value of this function when $x = 0$ we have $D/2\pi$. The function also has auxiliary maxima which may be disregarded in most cases. The value of the function in the first auxiliary maximum when $x D = 3\pi$ is approximately 5 times less than in the principal maximum. The half-width of the principal maximum (from the center of the maximum to zero) is, according to (15), equal to

$$\Delta x \approx 2\pi/D.$$

It is interesting to note that in the case of an infinite surface, as is evident from (14, 1) and (21, 1) and the expressions (32, 1) and (33, 1) analogous thereto, that, in the case of electromagnetic [waves], when $k_z^{in} \rightarrow 0$, the amplitude of the corresponding spectrum increases infinitely. In the case of a finite area, the amplitude of the spectrum, in accordance with (10), is proportional to $B_m(\alpha) \sin \alpha$, and, by virtue of the fact that $\alpha \rightarrow 0$ when $k_z \rightarrow 0$, the amplitude is finite.

The above-obtained results are valid for sufficiently remote points lying within the Fraunhofer zone where the condition

$$R \gg k D^2 \quad (16)$$

is satisfied. This condition is well known in optics. This comes about because of the requirement for slowness in changing the function $\Phi(k_z^0 + \pi p - \cos \alpha)$ as compared to the exponential curve in (8), which is necessary in order for it to be possible to move this function beyond the integral sign.

2. Diffraction on a Sinusoidal Surface

When examining specific examples we shall limit ourselves to the simplest case of a one-dimensional irregularity, assuming that function Z depends only on X . In the case of an infinite surface, the diffracted wave will consist of the aggregate of the flat waves, the directions of distribution of which will be set by the components of the wave vector k_m . In the case of a finite area of width D , the diffracted wave will be a cylindrical wave with a characteristic of directionality consisting of the aggregate of the lobes, the maxima of which lie in directions k_m . The angular half-width of the lobes is given by formula (13). The amplitude of the wave in the center of the lobe, according to (10) and (15), is given by:

$$A_m = (D/2\pi) \sqrt{\pi k/R} B_m(\alpha_m) \sin \alpha_m. \quad (17)$$

In the acoustic case $B_m(\alpha_m)$ is found from (21, 1). In the electromagnetic case, when the horizontal polarization is ($E_y \neq 0$, $E_x = E_z = 0$), $B_m^y(\alpha_m)$ is calculated from (33, 1) if it is assumed there that $q = 0$ at the same time that $B_m^x = B_m^z = 0$. We shall note that here $B_m^y(\alpha_m)$ coincides with $B_m(\alpha_m)$ for the acoustic case if V is taken to be -1 in (21, 1) -- as should have been expected. In the same manner, in the case of vertical polarization ($H_y \neq 0$, $H_x = H_z = 0$), for a magnetic field we shall obtain the formula (21, 1), where it is necessary to take V to be equal to 1. However, since in the future we will be interested only in the amplitudes of the waves and not in their phases, the cases of $V = 1$ and $V = -1$ are fully equivalent.

Thus the function

$$f(\alpha_m) = |B_m(\alpha_m)| \sin \alpha_m \quad (18)$$

characterizes the angular dependency of the amplitude of the scattered waves for the plane problem of both acoustic and electromagnetic waves. Making the exchange in (21, 1)

$$\begin{aligned} k_x^m &= k \sin \alpha_m, & k_x^0 &= k \cos \alpha_m, \\ k_z^0 &= -k \sin \alpha_0, & k_z^m &= k \cos \alpha_0 \end{aligned}$$

(where α_0 is the slide angle of the incident wave), and having assumed that $|V| = 1$, we obtain:

$$f(\alpha_m) = \frac{1 - \cos(\alpha_m - \alpha_0)}{\sin \alpha_m + \sin \alpha_0} |B_m^0(\alpha_m)|, \quad (19)$$

where, according to (17, 1), $B_m^0(\alpha_m)$ are the coefficients of expansion of the function:

$$\exp\{-ik(\sin \alpha_m + \sin \alpha_0)Z(X)\} = \sum_{m=-\infty}^{+\infty} B_m^0(\alpha_m) e^{im\rho X}. \quad (20)$$

We shall switch to an examination of scattering from a sinusoidal surface: in this case we have

$$Z = a \cos \rho X. \quad (21)$$

As is known, there occurs the relationship

$$e^{-ik \cos \rho X} = \sum_{m=-\infty}^{+\infty} (-i)^m J_m(s) e^{im\rho X}, \quad (22)$$

where J_m is a Bessel function of order m . Therefore, having designated

$$s \equiv ka(\sin \alpha_m + \sin \alpha_0) \quad (23)$$

and compared (20) and (22), upon calculating (19) we obtain

$$B_m^0(\alpha_m) = (-i)^m J_m(s). \quad (24)$$

Substituting (24) into (20), we obtain the final formula for wave amplitude in the maximum leaves, with an accuracy of up to the constant factor:

$$f(\alpha_m) = \frac{1 - \cos(\alpha_m - \alpha_0)}{\sin \alpha_m + \sin \alpha_0} J_m(s). \quad (25)$$

Figures 1-3 present graphs of the directionality characteristics of

a diffracted wave for two incident wave slide angles (45° and 10°) when there are various values for $\delta = 2\pi a/\lambda$, where a is the amplitude of the sinusoid, and λ is the length of the radiation wave. The length of the wave of sinusoid Λ is the same in all cases and is equal to $\Lambda = 10\lambda$.* However, the width of the lobes, which only the area dimension influence, is done very approximately on the graphs. A view of the uneven surface is depicted in the lower part of each Figure. Where the amplitudes of sinusoid are very low, the wavy line is depicted as a straight line. The arrows on the graphs indicate the directions of the incident and reflected wave where specular reflection exists. The force of the sound in decibels is indicated along the radius in all the graphs.

*The length of the area is taken as equal to 4Λ .

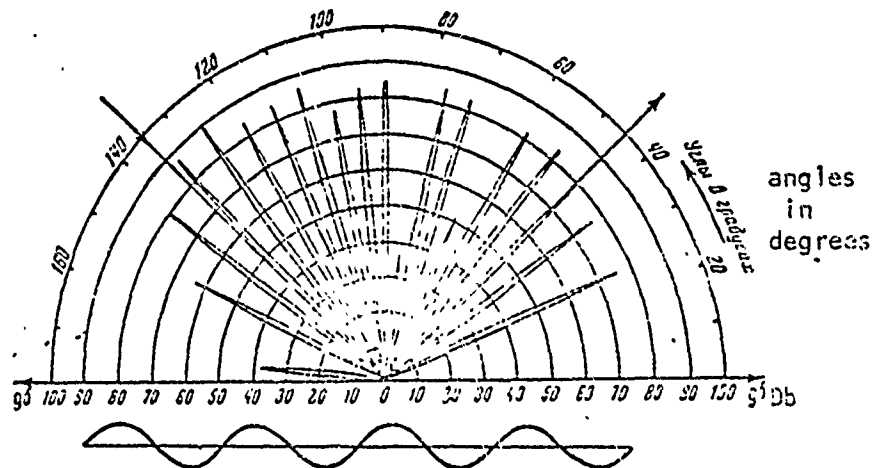
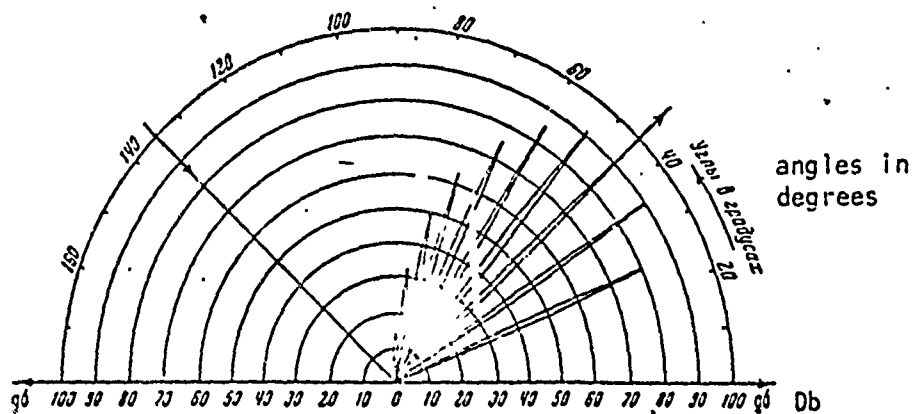
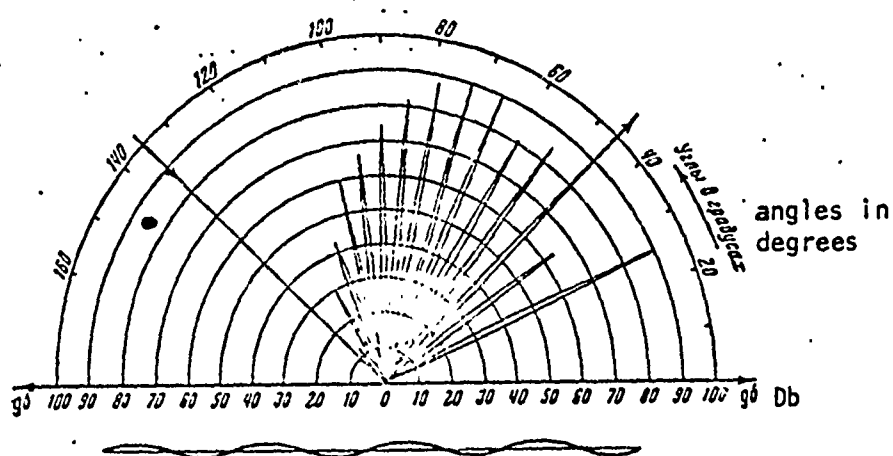
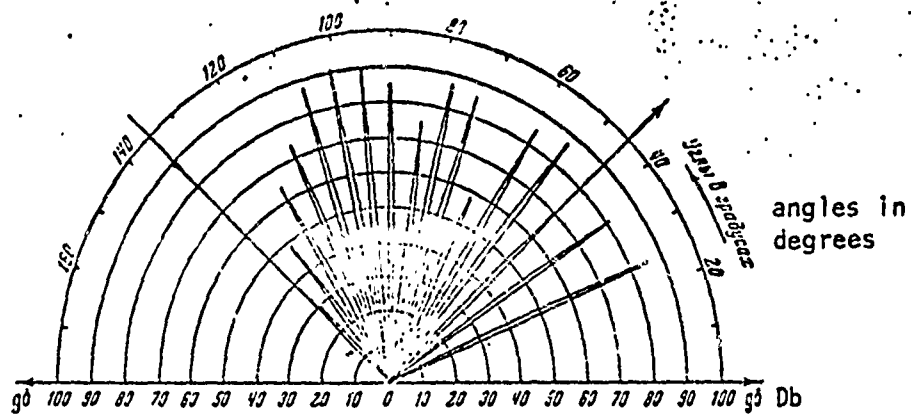


Figure 1: $\delta = 2\pi a/\lambda = 10$; $\Lambda/\lambda = 6.28$

We see that even at relatively low sinusoid ($\delta = 10$, i.e., $a = 1.6\lambda$, Figure 1) diffusion scattering almost occurs. Even at low amplitudes, when $\delta = 6$ and $\delta = 3$, maximum radiation scattering occurs not in the direction of specular reflection, but at angles equal to 100° and 72° , correspondingly. This is easy to understand, using the idea of rays. Rays specularly reflected from the point of inflection of the sinusoid go in these directions. In view of the fact that the curvature of the surface is equal to zero at the points of inflection, the rays, upon reflection, experience the least separation, as a consequence of which the amplitude of the corresponding waves is maximum.

The situation where a wavy surface, even one differing very little



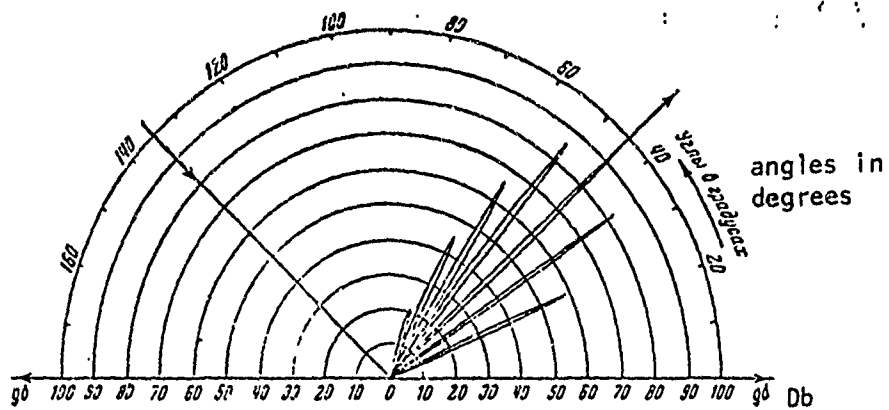


Figure 5: $\delta = 2\pi a/\lambda = 0.3; \Lambda/a = 209$

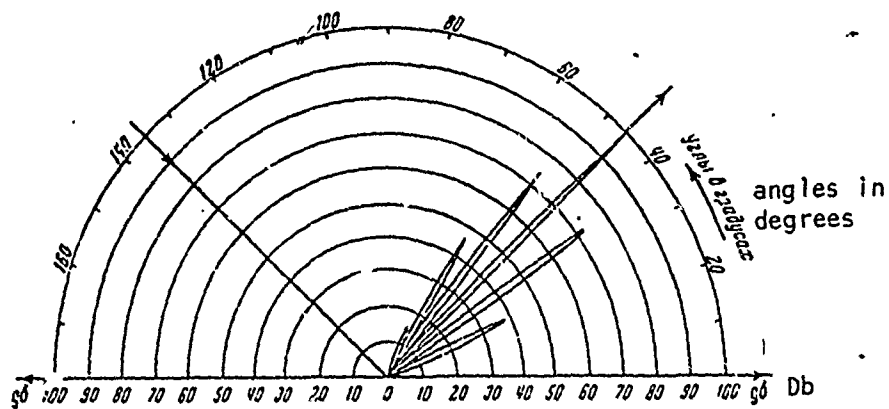


Figure 6: $\delta = 2\pi a/\lambda = 0.1; \Lambda/a = 628$

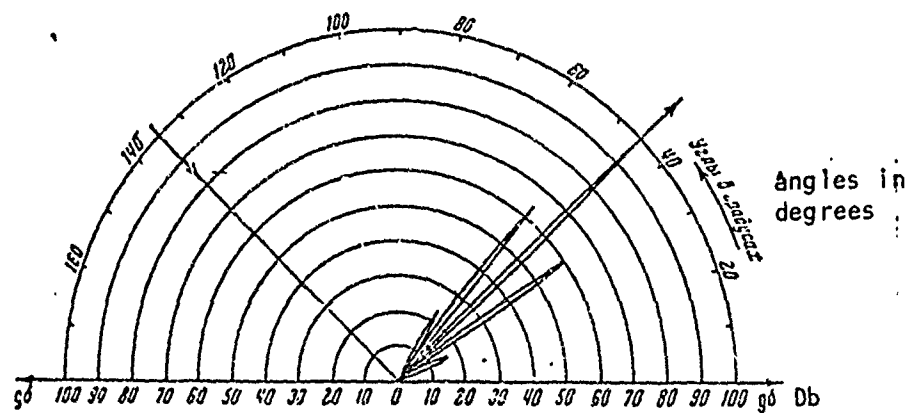
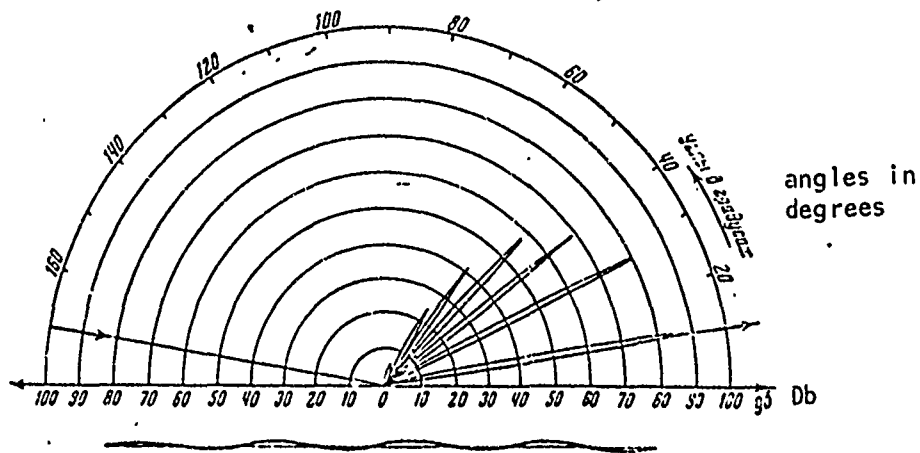
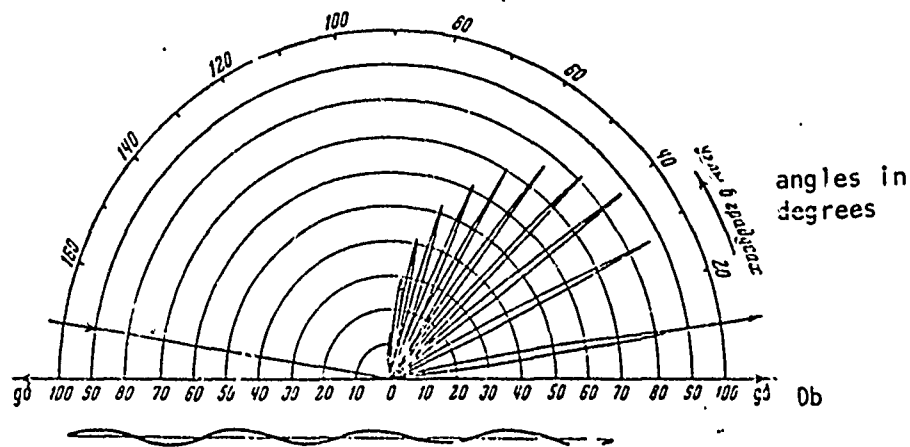
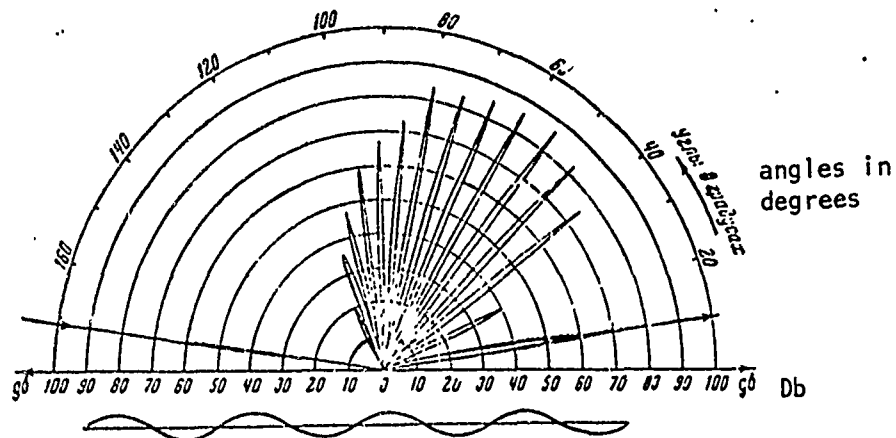


Figure 7: $\delta = 2\pi a/\lambda = 0.03; \Lambda/a = 2090$



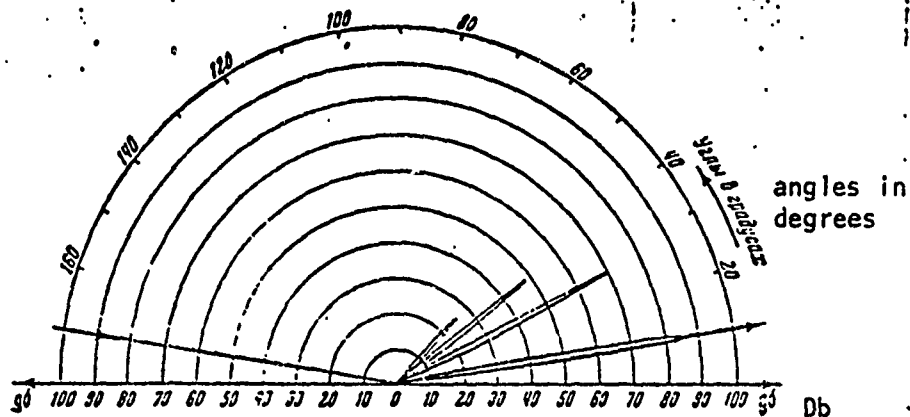


Figure 11: $\delta = 2\pi a/\lambda = 0,3$; $\Lambda/a = 209$

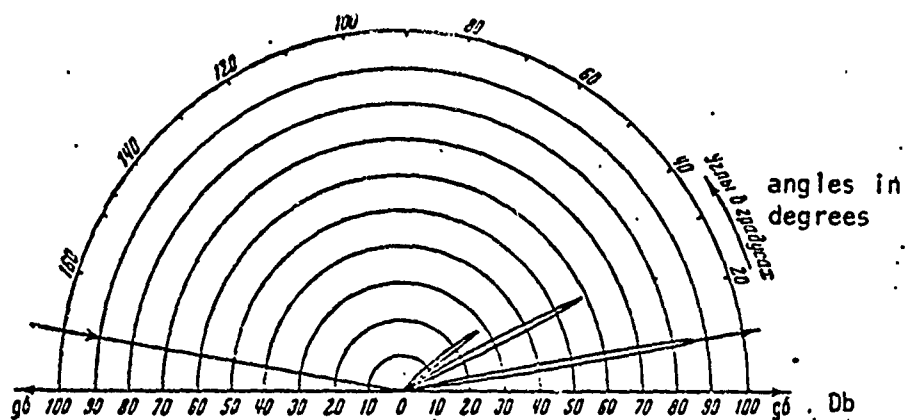


Figure 12: $\delta = 2\pi a/\lambda = 0,1$; $\Lambda/a = 628$

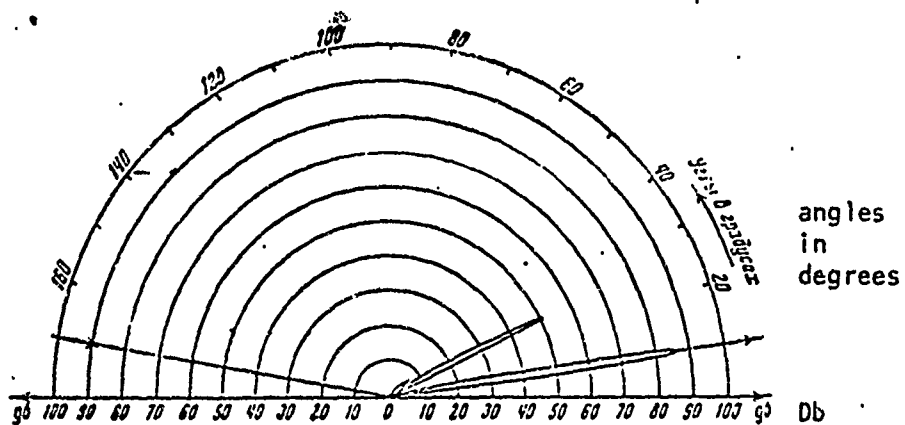


Figure 13: $\delta = 2\pi a/\lambda = 0,03$; $\Lambda/a = 2000$

from flatness, yields a noticeable deviation from specular reflection, as is evident, for example, from Figures 5 and 6, is interesting. Only in Figure 7, where the amplitude of the sinuosity is 209 times less than the length of the radiation wave and 2090 times less than Λ , may the reflection be considered specular, since the side lobes are approximately 30 db less than the principal lobe.

Where the slide angle of the incident wave is 10° , the scattering is more directional (compare, for example, Figures 3 and 9), given the same δ values, than is the case at 45° . This is a confirmation of the well-known fact that at low slide angles the reflection from a rough surface is more directional.

We shall also note that in all cases to which the above-presented figures apply, the theory applicability conditions indicated in § 5, 1 are satisfied. Specifically, the directions of the lobes depicted in the Figures nowhere greatly differs from the directions of possible geometric reflections of the rays.

3. A Comparison of Sinusoidal Surface Solutions Obtained by Various Methods

In the case of a sinusoidal surface one may obtain a solution to the problem by another method. Although the analysis of this solution is most complicated, it is useful to compare it to our solution for the case where the amplitude of the sinusoid is small in comparison to wave length.

For the sake of simplicity we shall examine the case of a wave striking the surface perpendicularly. We shall again write the equation for the surface in the form

$$Z = a \cos pX.$$

We shall assume the surface to be a reflecting surface, assuming that the boundary condition $\varphi = 0$ is fulfilled thereon. In electrodynamics this case corresponds absolutely to the conducting surface and to the vector E which is directed parallel to the y axis.

We shall write the total field of the incident and diffracted waves in the form²

$$\varphi = e^{-ikz} + A_0 e^{ikz} + A_1 e^{i\mu_1 z} \cos px + A_2 e^{i\mu_2 z} \cos 2px + \dots, \quad (26)$$

where

$$\mu_n \equiv \sqrt{k^2 - n^2 p^2}, \quad n = 1, 2, \dots$$

² Our method of obtaining a precise solution is essentially a generalization of Reiley's discussions [5].

This kind of full field representation coincides fully with the separation of the diffracted field into spectra [see, for example, formula (14, 1)] obtained in [1]. Here each term of the formula (26), beginning with the third, corresponds to two spectra, which is easy to envision if the cosines are expressed by exponential functions.

Substituting the surface equation $Z = a \cos pX$ into equation (26) and letting $\varphi = 0$, we obtain:

$$A_0 + \exp\{-2iak \cos pX\} + A_1 \exp\{-is_1 \cos pX\} \cos pX + A_2 \exp\{-is_2 \cos pX\} \cos 2pX + \dots = 0, \quad (27)$$

where

$$s_n = (k - \sqrt{k^2 - n^2 p^2}) a. \quad (28)$$

In equation (27) the exponential curves may be replaced by sums, if one considers that, analogously to (22), we have

$$e^{-is \cos pX} = J_0(s) - 2iJ_1(s) \cos pX - 2J_2(s) \cos 2pX + \dots$$

In the expression obtained after this [operation] the products of the cosines must be expressed by the cosines of the multiple angles. It is also necessary to group the terms containing the cosines of identical arguments as factors. Equating the coefficients before these cosines to zero, we obtain the following infinite system of equations for determining the coefficients A_0, A_1, \dots ³

$$\sum_{n=0}^{\infty} \gamma_{mn} A_n = C_m, \quad m = 0, 1, 2, \dots, \quad (29)$$

where

$$C_0 = J_0(2ak), \quad \gamma_{00} = 1, \quad (30)$$

$$C_m = 2(-i)^m J_m(2ak), \quad \gamma_{m0} = 0, \quad m = 1, 2, \dots$$

and finally

$$\gamma_{mn} = (-i)^{m+n} [J_{n+m}(s_n) + (-1)^m J_{n-m}(s_n)], \quad m, n = 1, 2, \dots \quad (31)$$

Investigation of the system of equations (29) is an extremely complicated problem. It would be of interest, first and foremost, to investigate the convergence of the series (26) where the coefficients A_0, A_1, A_2, \dots are to be found from (29). We shall limit ourselves here to an examination of the case of small irregularities ($ak \ll 1$),

³ We omit here computations reduced principally to a transformation of the order of summing in compound sums.

when the coefficients decrease very rapidly as n increases [as $(ak)^n$], and we shall compare the results obtained with the results of the above-presented theory. Retaining in the expressions for A_n only the lowest powers of ak , we find from (29) that

$$\begin{aligned} A_0 &= -1, \quad A_1 = 2iak, \quad A_2 = (ak)^2 \left[1 - \frac{s_1}{2ak} \right], \\ A_3 &= -i(ak)^3 \left[\frac{1}{3} - \frac{1}{2} \left(\frac{s_1}{2ak} \right)^2 - \frac{s_2}{2ak} \left(1 - \frac{s_1}{2ak} \right) \right], \\ &\dots \end{aligned} \quad (32)$$

Changing (26) to the form of (14, 1) by means of substituting exponential curves for cosines, one may convince oneself that $A_0, A_1, A_2, A_3, \dots$ must coincide with the values for $B_0, 2B_1, 2B_2, 2B_3, \dots$. The latter may be found according to formulas (26, 1) and (24). Here it is necessary to take into account the fact that $V = -1$ with our limited conditions. Besides that, since the incidence is assumed to be perpendicular, $k_z^0 = -k$, $k_x^0 = 0$. As a result, we obtain

$$B_m = -(k/k_z^m) (-i)^m J_m [ak (\sin \alpha_m + \sin \alpha_0)] \quad (33)$$

and, finally, considering the smallness of ak as well as the relationships $k_z^m = k \sin \alpha_m$ and $\alpha_0 = \pi/2$, we find that

$$\begin{aligned} B_0 &= -1, \quad 2B_1 = \frac{iak}{\sin \alpha_1} (1 + \sin \alpha_1), \quad 2B_2 = \frac{1}{\sin \alpha_2} \left(\frac{ak}{2} \right)^2 (1 + \sin \alpha_2)^2, \\ 2B_3 &= \frac{1}{3 \sin \alpha_3} \left(\frac{ak}{2} \right)^3 (1 + \sin \alpha_3)^3. \end{aligned} \quad (34)$$

Here

$$\sin \alpha_m = \frac{k_z^m}{k} = \sqrt{1 - \frac{m^2 p^2}{k^2}}, \quad m = 1, 2, 3.$$

Comparing the sequences of (32) and (34) we see that they will coincide, if we disregard $(mp)^2$ in comparison with k^2 . As a matter of fact, here, on one hand, $\sin \alpha_m \approx 1$, and, on the other hand, in conformity with (28), $s_m \approx 0$ and we obtain:

$$\begin{aligned} A_0 &= B_0 = -1, \quad A_1 = 2B_1 = 2iak, \\ A_2 &= 2B_2 = (ak)^2, \quad A_3 = -(i/3)(ak)^3 \dots \end{aligned} \quad (35)$$

Thus the validity of our solution in the case being examined

is limited by the condition that $(mp)^2 \ll k^2$, which is equivalent to the condition that $\sin \alpha_m = 1$, i.e., $\alpha_m \approx \pi/2$. In other words, we shall obtain correct amplitudes only for waves scattered in directions close to perpendicular. This result confirms the conviction expressed in § 5.1 above that the method of calculation is valid only for waves scattered in directions close to the directions of geometric reflection from various parts of the surface. In the case being examined, in view of the smallness of the amplitudes of the sinusoid, the directions of geometric reflection will be grouped around the perpendicular to the average surface level.

4. Wave Superposition

We shall examine the case where a wavy surface is formed by superpositioning several waves.

We shall assume that we have solved the problem of diffraction on a surface, the form of which is given by a certain function $Z(X)$. This means that the coefficients of $B_m^0(\alpha_m)$ have been found in an expansion of (20). The question is asked, how do these coefficients, and, consequently, the amplitudes of the diffracted spectra as well, change if a sinusoid defined by the cosine curve $a \cos \gamma p X$ and having a period γ whole number of times less than the period of function $Z(X)$ is superimposed on this surface?

We shall designate the new coefficients which are analogous to $B_m^0(\alpha_m)$ as $C_m^0(\alpha_m)$. The latter may be found from an expression which is analogous to (20):

$$\exp \{-ik(\sin \alpha_m + \sin \alpha_0)[Z(X) + a \cos \gamma p X]\} = \sum_m C_m^0(\alpha_m) e^{im p X},$$

which, when (20) and (23) are computed, is written in the form

$$e^{-is \cos \gamma p X} \sum_{n=-\infty}^{+\infty} B_n^0 e^{in p X} = \sum_{m=-\infty}^{+\infty} C_m^0 e^{im p X}.$$

Substituting the series (22) for the exponential curves in the left-hand portion, multiplying out the series and grouping terms correspondingly, we obtain

$$\sum_{n=-\infty}^{+\infty} e^{in p X} \sum_{k=-\infty}^{+\infty} (-i)^k J_k(s) B_{m-k}^0 = \sum_{m=-\infty}^{+\infty} C_m^0 e^{im p X},$$

from whence

$$C_{n1}^0(\alpha_m) = \sum_{k=-\infty}^{+\infty} (-i)^k J_k(s) B_{m-k}^0(\alpha_m). \quad (36)$$

Figure 14 depicts the scatter field in the case where the sinuosity is a superpositioning of two sinusoids. Calculation is accomplished with the help of formula (36). The angle of wave incidence is taken to be 45° . Besides that, it is taken that

$$\delta_1 = ka_1 = 3, \text{ and } \delta_2 = ka_2 = 2,$$

where a_1 and a_2 are the amplitudes of the sinusoid. As in the preceding cases, it is assumed that $\Lambda/\lambda = 10$.

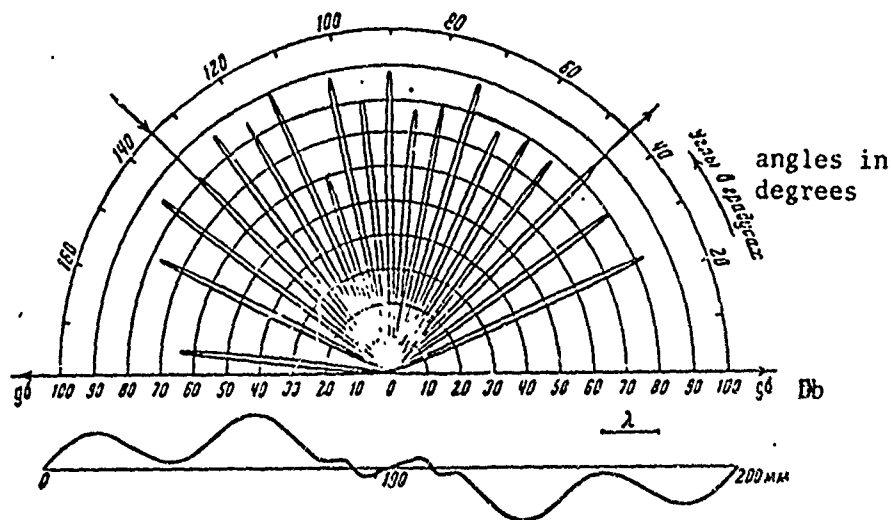


Figure 14: $\delta = 3, \delta_1 = 2$

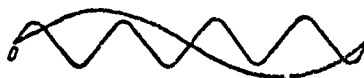


Figure 15: Individual sinusoids comprising the surface shown in Figure 14

A view of the scattering surface is depicted in the lower portion of the Figure [14]. Individual sinusoids, the superposition of which results in this surface, are depicted in Figure 15.

5. Diffraction on a surface which, [when viewed] cross-sectionally, is a broken line

We shall examine diffraction on a surface defined by the equations (Figure 16):

$$\begin{aligned} Z &= a(1 + 4X/\Lambda), & -\Lambda/2 \leq X \leq 0, \\ Z &= a(1 - 4X/\Lambda), & 0 \leq X \leq \Lambda/2. \end{aligned} \quad (37)$$

For the expansion coefficients of (20) we have, according to the known formulas for Fourier series coefficients,

$$B_n(\alpha_n) = \frac{1}{\Lambda} \int_{-\Lambda/2}^{\Lambda/2} \exp \{-ik(\sin \alpha_n + \sin \alpha_0)Z(X) - impX\} dX \quad (38)$$

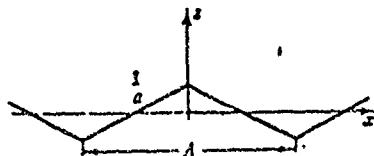


Figure 16: Cross-sectional view of a "broken-line" surface or, taking into account (37) and (23),

$$\begin{aligned} B_n(\alpha_n) &= \frac{1}{\Lambda} \int_{-\Lambda/2}^0 \exp \left\{ -is \left(1 + \frac{4X}{\Lambda} \right) - impX \right\} dX + \\ &+ \frac{1}{\Lambda} \int_0^{\Lambda/2} \exp \left\{ -is \left(1 - \frac{4X}{\Lambda} \right) - impX \right\} dX. \end{aligned}$$

Having completed the integration, we obtain

$$B_n^0(\alpha_n) = \frac{i^{m+1}s}{\left(\frac{m\pi}{2}\right)^2 - s^2} \sin \left(\frac{m\pi}{2} + s \right), \quad (39)$$

where the relationship $p = 2\pi/\Lambda$ is also taken into account.

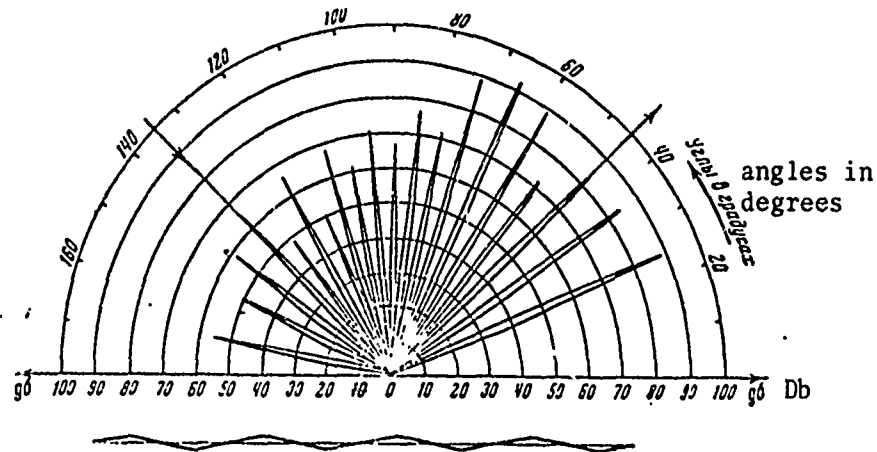


Figure 17: Scattered wave directionality characteristic according to formula (40)

$$\delta = 2\pi a/\lambda, \Lambda/a = 20.9$$

Having substituted (39) into (19), we find for the amplitudes of the waves at the lobe maxima with an accuracy up to a constant factor that:

$$f(\alpha_m) = \frac{ak[1 - \cos(\alpha_m - \alpha_0)]}{\left(\frac{m\pi}{2}\right)^2 - s^2} \sin\left(\frac{m\pi}{2} + s\right). \quad (40)$$

Figure 17 depicts the scattered wave directionality characteristic calculated according to formula (40). The angle of incidence is taken to be 45° , $\Lambda/\lambda = 10$, and $\delta = 2\pi a/\lambda = 3$.

As may be seen, the scattered wave has a character close to that of diffusion. The lobe directed at an angle of approximately 66° has maximum amplitude. This lobe corresponds to specular reflection from the flat areas forming the left slopes of each peak in the uneven surface (for example, the sector $-\Lambda/2 \leq X \leq 0$ in Figure 16).

The lobe directed at an angle of approximately 23° and corresponding to reflection from the right slopes is also most intensive.

As was already indicated above (see § 5, 1), our theory is able

to give correct values for lobe magnitudes only for angles which do not differ too greatly from these two directions of geometric reflection. At angles greater than 90° it may not be expected that the dimensions of the lobes will be correct. They are depicted in Figure 17 only to illustrate the results of the theory, which is based on Kirchhoff's principle.

The scatter diagrams presented here were constructed by Ye. Zezyukina; she also did all the necessary calculations. In connection with this, I express my gratitude to her.

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